Thermal Analysis of the Drink Cooler Design

Cooling beverages using Peltier Cells represents a change from previous cooling methods. The existing product, the Cooper Cooler Blitz Chiller, used a circulating ice water bath to cool a can or bottle via forced convection. The product took advantage of the large amount of energy required to melt ice into water (the latent heat of fusion, or melting energy, for ice is 334,000 Joules/Kilogram), to keep a water bath at or near 0° Celsius. The cold-water bath was then sprayed over a rotating can to cool the beverage from room temperature to serving temperature.

The innovation of our product is to eliminate the need of purchasing or making ice to create an ice bath at or near 0° Celsius. By cooling the water bath to the same or comparable temperatures as the existing Cooper Cooler Blitz Chiller, the heat transfer between the can and sprayed water would remain the same. Thus the cooling time would remain constant and the major goal of eliminating use of ice from the product would be achieved. To accomplish this task, Peltier Cells were used to remove heat from the water.

Peltier Cells utilize the Peltier Effect to create a temperature differential from an electrical differential as heat is forced from one side of the cell to the other. As current is run through the Peltier Cell, one side becomes extremely hot and one side becomes extremely cold. As the Cell operates, heat must be removed or the Cold side of the cell will begin to warm due to conductive heat transfer through the Peltier Cell.

To cool the water, two Peltier Cells are used to chill an aluminum and copper heat exchanger assembly.

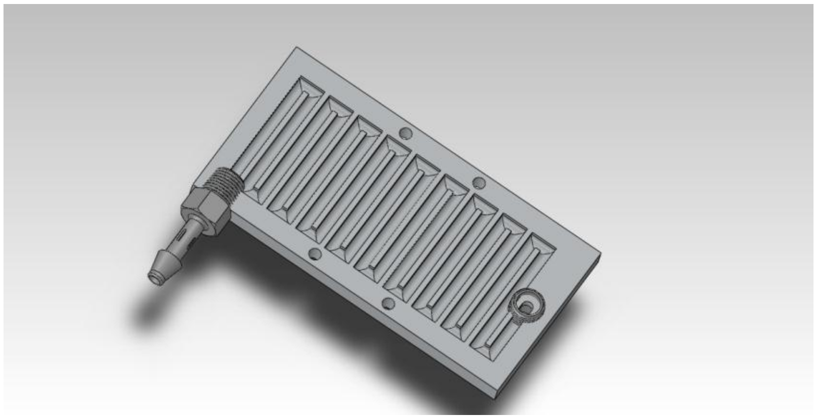


Figure 1: Aluminum Heat Exchanger Flow Channel Block

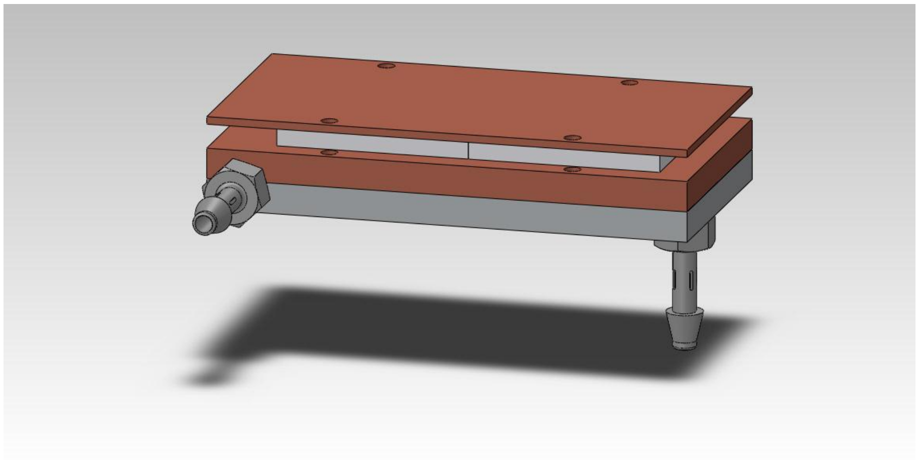


Figure 2: Heat Transfer Block Assembly

Water from the cooling bath is pumped through the flow channels of aluminum plate. A copper plate is bolted to the top of the aluminum flow channels to seal the block. Two Peltier Cells are mounted to the top of the copper plate. The cold side of the Peltier Cell cools the copper plate and the aluminum flow channels and chills the water bath as it flows through the channels. The hot side of the Peltier is attached to another copper plate. This second copper plate is attached to a commercial CPU heat sink that is used to remove heat from the two Peltier Cells.

To analyze the thermal performance of our product, we will focus on and analyze four main heat transfer challenges in our product. The first challenge is the capability of the chilled heat transfer channels to cool the water bath being pumped through it. The second challenge is the ability of the Peltier Cell to cool the aluminum and copper heat transfer block.
The third is the ability to manage heat produced in the Peltier Cell during operation. The fourth challenge is the ability of the copper plate and heat sink to remove heat from the Peltier Cells and transfer it into the ambient environment. By examining the heat transfer at each of these three zones, it is possible to determine the cooling capability of the product.

Zone 1: Heat Transfer from the Cooling Channels to the Water Bath

Inside of the Aluminum/Copper cooling block, water flows through at a constant rate (due to the steady state operation of the motor) and is cooled by the cold walls of block.

To calculate the heat transfer from the cooling block to the water bath, a convective heat transfer model can be used:

$$q = \bar{h}\, A_s (T_s - T_\infty) \qquad \text{Equation 1}$$

Where q is the total heat transfer rate, $\bar{h}$ is the average convective heat transfer coefficient, A_s is the surface area of heat transfer, T_s is the surface temperature, and T_∞ is the ambient temperature of the fluid.

The variable of A_s can be controlled by the design of the flow block and channels and $\bar{h}$ can also be controlled by the design the flow channels and by controlling the flow of water through the system. The variable of T_∞ is a transient property that will change based on temperature of the cooling bath flowing through the flow block while the variables of q and T_s are related by the performance of the Peltier Cell at different operating amperages.

In order to maximize the total heat transfer rate shown in Equation 1, the system should be engineered to maximize $\bar{h}$ and A_s, and to maintain the smallest possible T_s.

To first analyze the system, $\bar{h}$ needs to be derived. The convective heat transfer coefficient, $\bar{h}$, can be derived based on the type of flow occurring in the pipe: laminar or turbulent. This regime of flow is determined by the flow of water in the channels, the properties of the water flowing, and the pipe geometry, as represented by the non-dimensionless Reynolds Number which relates viscous forces in a flow to inertial forces. The equation for the Reynolds number is:

$$Re = \frac{\rho \mathrm{V} d}{\mu} = \frac{\mathrm{V} d}{\nu} \qquad \text{Equation 2}$$

Where ρ is the fluid density, V is the flow velocity, d is the hydraulic diameter of the pipe, μ is the fluid viscosity, and ν is the kinematic viscosity.

To derive the hydraulic diameter of the flow channel, the following equation was used:

$$d = \frac{4\,A_c}{P} \qquad \text{Equation 3}$$

Where A_c is the cross section of the flow channel and *P* is the perimeter of the flow channel. To find the velocity of the water in the channels, the volumetric flow of water in the channels was experimentally found and the mean velocity was derived using the following equation:

$$\mathrm{V} = \frac{\dot{V}}{A} \qquad \text{Equation 4}$$

Where V is the average flow velocity through the heat sink, $\dot{V}$ is the volumetric flow rate supplied by the system pump, and *A* is the cross sectional area of the flow channels.

Thus the combined equation for the Reynolds Number is:

$$Re_{Cooling\ Block} = \frac{\rho\left(\frac{\dot{V}}{A}\right)\left(\frac{4\,A_C}{P}\right)}{\mu} = \frac{4\,\dot{V}_{channel}}{\nu_{water}\,P_{channel}} \qquad \text{Equation 5}$$

Where *Re* is the Reynolds Number, V is the average fluid velocity, *P* is the perimeter of the channel, and *v* is the kinematic viscosity of the water. The values for the terms used and the calculation of the Reynolds number for the channel, $Re_{Channel}$, can be found in Appendix A.

Using the three equations above, the Reynolds number was calculated to be greater than 10000 so the flow in the channels could be considered turbulent. The Dittus Boelter equation is a correlation that relates the Reynolds number of a flow and the Prandlt number of a fluid (a non dimensional number that relates momentum diffusivity to thermal diffusivity) to the Nusselt number (a non dimensional number that relates conductive heat transfer to convective heat transfer in the fluid). The Dittus Boelter equation is:

$$\overline{Nu}_d = 0.023\,Re_D^{4/5}Pr^{0.3} \qquad \text{Equation 6}$$

Where Re is the Reynolds number and Pr is the Prandlt number.

The Nusselt number can then be used to derive $\bar{h}$ using the following correlation:

$$\overline{Nu}_d \equiv \frac{\bar{h}D}{k} \qquad \text{Equation 7}$$

Where D is the hydraulic diameter of the pipe and k is the thermal conductivity of the fluid. Thus, the final equation for the convective heat transfer coefficient in the flow channels for turbulent flow is:

$$\bar{h} = 0.023\,\frac{k}{D}\,Re_D^{4/5}Pr^{0.3} \qquad \text{Equation 8}$$

$$\bar{h}_{Cooling\ Block} = 0.023\,\frac{k_{Water}}{\frac{4\,A_{channel}}{P_{channel}}}\left(\frac{4\,\dot{V}_{channel}}{\nu_{water}\,P_{channel}}\right)^{4/5} Pr_{Water}{}^{0.3} \quad \text{Equation 9}$$

The values of the terms used and the calculations of the convective heat transfer coefficient in the channel, $\bar{h}_{channel}$, can be found in Appendix A.

Thus, by using the above equations, if the surface temperature or heat transfer rate is known, we can solve for the other using Equation 1. These boundary condition are correlated via the heat transfer equations in Zone 2.

Zone 2: Heat Transfer from the Peltier Cell through the Copper Plate

The movement of heat from one side of the Peltier Cell to the other is the driving cooling force in the product. Heat is transferred through the copper plate via conduction. The heat transfer via conduction can be modeled using the following equation:

$$q = \frac{kA}{L}\,(T_h - T_c) \qquad \text{Equation 10}$$

Where q is the total heat transfer rate, k is the thermal conductivity of the material, L is the thickness of the material, A is the heat transfer surface area of the plate, T_h is the hot side temperature of the plate, and T_c is the cold side temperature of the plate.

As shown by Equation 10, the heat transfer is a function of the temperature difference between the two sides of the plate (T_h and T_c), the plate's material properties (k), and the plate's geometry (L, A). T_h and T_c will be determined by system boundary conditions, but the parameters of L, A, and k can all be controlled. To maximize q, a copper plate was used because of its high thermal conductivity (k), and designed to have large surface area (A), and a small thickness (L).

In Zone 2, T_h represents the temperature of the cooling channel surface and T_c represents the temperature of the surface of the Peltier. Thermal contact resistance, or the resistance to heat transfer caused by air small air pockets that exist between the surfaces of two rough, touching materials, has been ignored for this analysis. All thermal mated surfaces have you been coated with a highly thermally conductive silicon thermal paste to reduce contact resistance.

Based on Equation 10 and the assumption of negligible conductive contact resistance, if T_c(the Peltier Cold Side temperature) or the heat flux through the system is known known, the heat transfer through the block and into the fluid can be determined.

Zone 3: Heat Transfer and Heat Production in the Peltier Cell

Inside of the Peltier Cell, the Peltier Effect or Thermoelectric Effect produces a thermal differential from a temperature differential. The major challenge in using a Peltier Cell is managing the heat produced by the hot side of the Cell. If heat is not removed sufficiently fast enough, then the Peltier Cell will begin to heat, increasing the temperature of the cold side and decreasing cooling. The heat balance is represented by the following equation:

Heat Rate = Peltier Cooling Effect – Joule Heating – Heat Conduction from Hot Side

These three terms are governed by the symbolic equation for the cold side of the Peltier:

$$Qc \cong -S\, I\, T_c + \frac{1}{2}\, I^2 R + k\, \frac{A}{L}\, (T_h - T_c) \quad \text{Equation 11}$$

Where *Qc* is the heat transfer rates on the cold side of the Peltier Cell, *S* is the Seebeck Coefficient (a factor which indicates the effectiveness of the Peltier Cell at creating a temperature differential), *I* is the electrical current powering the Peltier Cell, T_c is the cold side temperature, *R* is the electrical resistance of the Peltier Cell (a term which will effect the amount of joule heating which occurs in the cell during operation), and a conductive heat transfer equation using the same variables as was introduced in Equation 3.

To maximize heat removal, the first term (the Peltier Cell Cooling Effect) must be maximized and the second two terms must be minimized. One of the major problems with our prototype was that we could not run it at full power and full cooling capacity due to the difficultly and complexity of building electronic circuits that run at high amperages. The Peltier Cells used in the prototype are designed to run at full power while using 9 amperes of current, but due to our team's inexperience with high amperage circuits and the lack of commercially available high amperage products (most commercial products have a fuse that will trip at 4-5 amps), we were only able to achieve a current of 3 amps across the Peltier Cells. Consequently, the prototype was not cooling the prototype to its full capacity.

Another major factor that controls the effectiveness of the Peltier Cells is the hot side temperature of the Peltier Cell. As the temperature of the hot side of the Peltier Cell increases, the overall temperature of the Peltier Cell will increase due to conductive heat transfer through the cell. By effectively cooling the hot side of the Peltier Cell, maximum cooling can be achieved on the cold side of the cell.

Zone 4: Heat Transfer through the Copper Plate and Heat Sink

The cooling of the hot side of the Peltier Cell is essential for operation of the prototype. In the prototype, a copper plate is attached to the two Peltier Cells to conduct heat from the surface and then is dissipated by a commercially made CPU fan-heat sink unit. To model the heat transfer through this section of the prototype, the heat transfer from the copper plate and fins of the heat sink will be approximated using conductive and convective heat transfer equations respectively.

Conduction through the copper plate on the hot side of the Peltier Cell can be modeled using Equation 3.

To calculate the convective heat transfer from the top of the copper plate, the area and convective heat transfer coefficient of the heat sink must be derived. To simplify the approximation, the heat sink was modeled as an array of flat plates held at the surface temperature. The extremely high thermal conductivity of the copper heat pipes of the heat sink that connect the aluminum fin array to the copper plate will keep the fins at a temperature close to that of the copper plate surface. To calculate the convective heat transfer coefficient the flow regime (laminar or turbulent) first needed to be determined. First, the velocity of the air supplied by the heat sink fan was calculated using the following equation:

$$\mathrm{V} = \frac{\dot{V}}{A} \qquad \text{Equation 4}$$

Where V is the average flow velocity through the heat sink, $\dot{V}$ is the volumetric flow rate supplied by the fan, and *A* is the cross sectional area of flow through the heat sink. Given the airflow velocity through the heat sink, the Reynold Number could be calculated using the following equation to determine the flow regime:

$$Re_{Heat\ Sink} = \frac{\left(\frac{\dot{V}_{fan}}{A_{heat\ sink}}\right) d_{heat\ sink}}{\nu_{air}} \qquad \text{Equation 5}$$

Where *Re* is the Reynolds Number, V is the average airflow velocity, d is the characteristic channel width, and *v* is the kinematic viscosity of the fluid.

Due to the geometry of the heat exchanger plates, the following approximation was used:

$$d = 2\,w \qquad \text{Equation 12}$$

Where *w* is the width between two parallel plates of the heat sink. The values for the terms used and the calculation of the Reynolds Number for the heat sink, $Re_{Heat\ Sink}$, can be found in Appendix A. The Reynolds number was far less than 2300 indicating that the flow through the heat sink was laminar. Given laminar flow across parallel plates, the flowing equation can be used to calculate the Nusselt Number and the convective heat transfer coefficient:

$$\overline{Nu}_L \equiv \frac{\bar{h}_L L}{k} = 0.664\, Re_L^{1/2} Pr^{1/3} \qquad \text{Equation 13}$$

Where L is the length of the plate and *Pr* is the Prandlt number, the non-dimensionless number that relates momentum diffusivity to thermal diffusivity. Thus, the derived form of the equation for the convective heat transfer coefficient in the heat sink is:

$$\bar{h}_{Heat\ Sink} = 0.664 \frac{k_{Air}}{L_{Heat\ Sink}} \left(\frac{2\dot{V}_{Fan} w_{Plate\ Spacing}}{\nu_{Air} A_{Heat\ Sink}}\right)^{1/2} Pr_{Air}{}^{1/3} \text{ Equation 14}$$

The values of the terms used and the calculations of the convective heat transfer coefficient in the channel, $\bar{h}_{Heat\ Sink}$, can be found in Appendix A.

Thus, now that all aspects of the heat transfer have been characterized, it is possible to create a thermal circuit to simply calculate the heat transfer through the system using information regarding Peltier Cell performance.

Modeling the Product Performance using Thermal Circuits

Thermal circuits are a common way to illustrate the flow of heat through a system and simplify the calculations that must be performed. In a thermal circuit, each junction node represents a temperature boundary shared by two different heat transfer modes. The resistors the join two nodes represent different heat transfer equations that characterize the heat transfer between two temperatures through a material or over a surface. In general, the thermal resistance of a heat transfer mode is equal to the ratio of the temperature difference to heat transfer, solved symbolically. The two main equations for thermal resistors that we will be using are the equations for conduction and convection. For conduction, the heat transfer equation and the thermal resistance model are shown below:

$$q = \frac{kA}{L}(T_h - T_c) \qquad \text{Equation 10}$$

$$R_{Conduction} = \frac{(T_h - T_c)}{q} = \frac{L}{kA} \qquad \text{Equation 15}$$

Where $R_{Conduction}$ is the thermal resistance of the conductive heat transfer, q is the total heat transfer rate, k is the thermal conductivity of the material, L is the thickness of the material, A is the heat transfer surface area of the plate, T_h is the hot side temperature, and T_c is the cold side temperature.

For convection, the heat transfer equation and the thermal resistance model are shown below:

$$q = \bar{h}\, A_s(T_s - T_\infty) \qquad \text{Equation 1}$$

$$R_{Convection} = \frac{(T_h - T_c)}{q} = \frac{1}{hA} \qquad \text{Equation 16}$$

Where $R_{Convection}$ is the thermal resistance of the conductive heat transfer q is the total heat transfer rate, $\bar{h}$ is the average convective heat transfer coefficient, A_s is the surface area of heat transfer, T_s is the surface temperature, and T_∞ is the ambient temperature of the fluid.

The advantage of calculating heat transfer using thermal resistances is that it becomes much simpler to take boundary conditions such as continuous heat flux and temperature into account because it becomes possible to sum across multiple resistors, much like in an electrical circuit. Boundary temperatures at the edges of a thermal circuit may be considered and resistances across the circuit summed. For example:

$$R_{Total} = R_1 + R_2 + R_3 + \cdots \qquad \text{Equation 17}$$

Where R_1 et al. are individual conductive or convective resistance terms. If the boundary conditions for the circuit are known (i.e. the temperatures at both ends), the following equation may be used to solve for the heat transfer rate through the entire system:

$$R_{Total} = \frac{(T_{Start} - T_{End})}{q} \qquad \text{Equation 18}$$

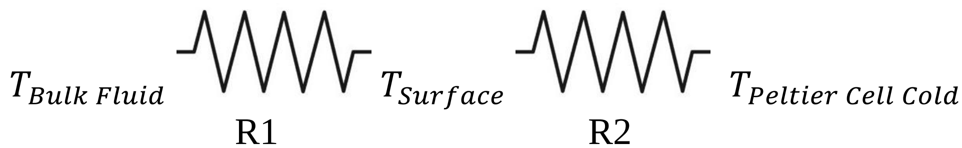
Where T_{Start} and T_{End} are the bounding temperatures of the circuit and q is the heat transfer rate through the system. Thus the heat transfer through the circuit would be:

$$q = \frac{(T_{Start} - T_{End})}{R_{Total}} \qquad \text{Equation 19}$$

This approach was used to calculate the heat transfer in the prototype. Heat transfer was calculated from the ambient air in the surroundings to the hot side of the Peltier Cell and then from the cold side of the Peltier Cell to the water flowing through the system. A Peltier Performance Chart provided by the cell manufacturer was then used to join the two analyses based on the supplied amperage to the Peltier Cell.

Calculating the Peltier Cell Cold Side Thermal Circuit

The thermal circuit on the cold side of the Peltier Cell (top of the Peltier Cell), was composed of the convective heat transfer term that related the heat transfer channel surface temperature and the bulk water temperature and the conductive term of heat transfer through the bottom copper plate that was connected to the cold side of the Peltier Cell. The thermal circuit for those two terms is shown below:



Where:

$$R1 = \frac{1}{h_{Cooling\ Block} A_{Cooling\ Block}} \quad \text{and} \quad R2 = \frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}}$$

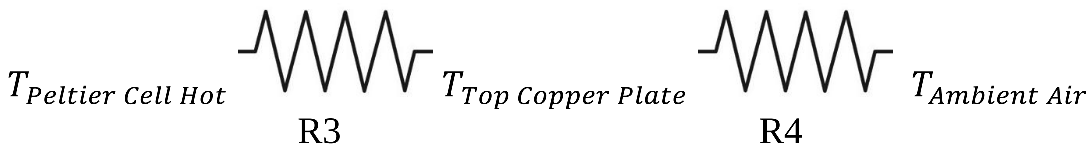
Using the Thermal Resistance models previously explained, the heat transfer on the cold side of the Peltier Cell can be expressed as:

$$q_c = \frac{(T_{Bulk\ Fluid} - T_{Peltier\ Cell\ Cold})}{\frac{1}{h_{Cooling\ Block} A_{Cooling\ Block}} + \frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}}} \qquad \text{Equation 20}$$

Thus, the heat transfer on the cold side of the Peltier Cell is fully characterized.

Calculating the Peltier Cell Hot Side Thermal Circuit

The thermal circuit on the cold side of the Peltier Cell (top of the Peltier Cell), was composed of the convective heat transfer term that related the heat transfer channel surface temperature and the bulk water temperature and the conductive term of heat transfer through the bottom copper plate that was connected to the cold side of the Peltier Cell. The thermal circuit for those two terms is shown below:



Where:

$$R3 = \frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} \quad \text{and} \quad R4 = \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}}$$

Using the Thermal Resistance models previously explained, the heat transfer on the cold side of the Peltier Cell can be expressed as:

$$q_h = \frac{(T_{Peltier\ Cell\ Hot} - T_{Ambient\ Air})}{\frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} + \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}}} \qquad \text{Equation 21}$$

Thus, the heat transfer on the hot side of the Peltier Cell is fully characterized.

Joining the Two Resistance Models

To join these two circuit models, a relationship between q_h and q_c must be derived. As described earlier, the hot side of the Peltier Cell is heated by a combination of heat being moved the Seebeck effect and Joule heating that takes place due to the resistance of the material. The exact heat generation rate at the surface of the hot side of the Peltier Cell is difficult to calculate due to its transient nature, but a conservative approach may be taken to give an upper bounding value for heat generation at the interface. By assuming the Peltier Cell acts as resistor that will only heat due to Joule heating, i.e. where 100% of applied current is transformed into heat, a bounding case for heat transfer can be derived. The equation for Joule heating is:

$$P = I^2 R \qquad \text{Equation 22}$$

Where *P* is the thermal power generated, *I* is the current, and *R* is the resistance of the Peltier Cell. By setting this value equal to q_h and, with all other variables known or derived except for $T_{Peltier\ Cell\ Hot}$, the value can be derived. The equation for $T_{Peltier\ Cell\ Hot}$ is then:

$$q_h = \frac{(T_{Peltier\ Cell\ Hot} - T_{Ambient\ Air})}{\frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} + \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}}} \qquad \text{Equation 21}$$

$$q_h \left(\frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} + \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}} \right) = (T_{Hot} - T_{Ambient})$$

Equation 23

$$T_{Hot} = q_h \left(\frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} + \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}} \right) + T_{Ambient}$$

Equation 24

$$T_{Hot} = I^2 R \left(\frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} + \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}} \right) + T_{Ambient}$$

Equation 25

Where $h_{Heat\ Sink} = 0.664 \frac{k_{Air}}{L_{Heat\ Sink}} \left(\frac{2\dot{V}_{Fan} w_{Plate\ Spacing}}{\nu_{Air} A_{Heat\ Sink}} \right)^{1/2} Pr_{Air}^{\ 1/3}$

Equation 14

With this equation, the maximum hot side temperature under steady state conditions is known for given amperage. The values for the terms in the equation above are given in Appendix A.

In Peltier Cells, the maximum temperature differential is a function of both the heat flux and the operating amperage. The chart below, provided by the manufacturer, displays the variable performance of the Peltier Cell:

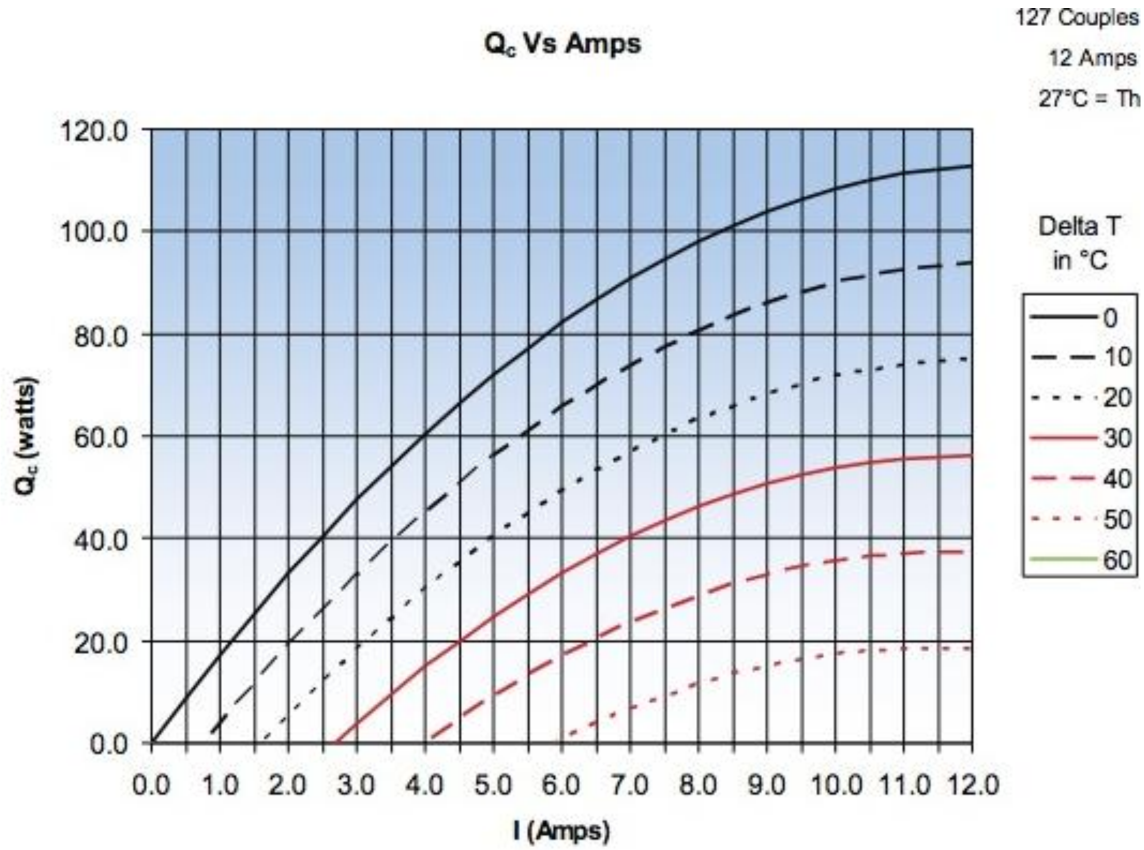


Figure 3. Chart of Peltier Cell Performance

Using the figure above and the supplied amperage to the Peltier Cell, it is possible to correlate the hot side Peltier temperature (solved for above) with the values of the cooling rate (Q_C) and the cold side temperature (T_c) solved for via the following equation:

$$T_c = T_h - \Delta T \qquad \text{Equation 26}$$

To test the effect of different amperages, temperature differentials, and cooling rates, 4 test cases are run in Appendix A to see what the maximum cooling rate would be given a certain temperature goal. For the four tests, the criteria was that the product would have to cool the cold side of the Peltier Cell to below 5°C, a drink temperature indicated on the high end of our users preferences for drink temperature in our user studies. The temperature differential needed, the T_c of the Peltier Cell and the cooling rate for each of the four cases are shown in Appendix A.

By using the cold side Peltier Cell heat transfer equation solved using thermal resistance model:

$$q_c = \frac{(T_{Bulk\ Fluid} - T_{Peltier\ Cell\ Cold})}{\frac{1}{h_{Cooling\ Block} A_{Cooling\ Block}} + \frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}}} \qquad \text{Equation 20}$$

It is now possible to characterize the final temperature of the Bulk Fluid in the Peltier Cell at steady state:

$$T_{Fluid} = q_c \left(\frac{1}{h_{Cooling\ Block} A_{Cooling\ Block}} + \frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} \right) + T_{Cold}$$

Equation 27

Where:

$$h_{Cooling\ Block} = 0.023 \frac{k_{Water}}{\frac{4 A_{channel}}{P_{channel}}} \left(\frac{4 \dot{V}_{channel}}{\nu_{water} P_{channel}} \right)^{4/5} Pr_{Water}{}^{0.3}$$

Equation 9

The values for the terms in the equation above are given in Appendix A for each of the four amperage tests.

Cooling Time for a Beverage

In our prototype, the heat transfer was characterized by four equations and the Peltier Cell performance chart. The four equations were the equations for the convective heat transfer coefficient for the Cooling Block and the Heat Sink and the equations for the boundary temperatures as derived by the thermal circuit analysis for the Cooling Block and the Heat Sink. Those equations are:

$$h_{Cooling\ Block} = 0.023\ \frac{k_{Water}}{\frac{4\ A_{channel}}{P_{channel}}} \left(\frac{4\ \dot{V}_{channel}}{\nu_{water}\ P_{channel}} \right)^{4/5}\ Pr_{Water}{}^{0.3}$$

$$T_{Fluid} = q_c \left(\frac{1}{h_{Cooling\ Block} A_{Cooling\ Block}} + \frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} \right) + T_{Cold}$$

$$h_{Heat\ Sink} = 0.664 \frac{k_{Air}}{L_{Heat\ Sink}} \left(\frac{2\dot{V}_{Fan} w_{Plate\ Spacing}}{\nu_{Air} A_{Heat\ Sink}} \right)^{1/2}\ Pr_{Air}{}^{1/3}$$

$$T_{Hot} = I^2 R \left(\frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} + \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}} \right) + T_{Ambient}$$

The two sets of equations are joined by a Peltier Cell performance chart that shows the relationship between the hot side and cold side temperatures, the Peltier Cell cold side heat flux, and the operating amperage of the Peltier Cell. By using these five bounding relationships, it is possible to find the cooling rate of the water at various water temperatures and amperages.

To determine the effectiveness of our product, the rate at which the product is able to remove energy from the working fluid when chilling a beverage and the total time that would be necessary to chill the beverage must be known. To derive this relationship, an energy balance can be performed on the system using the following equation:

$$Q = V\rho C_p \Delta T \qquad \text{Equation 28}$$

Where Q is the energy removed, V is the volume of water cooled, ρ is the density of water, C_p is the heat capacity of water, and ΔT is the temperature change of the water.

To find the rate at which water is cooled, the rate of energy removal of the Peltier Cells, q_c, can be used. The equation relevant to the heat transfer is:

$$t = \frac{Q}{q_c} \qquad \text{Equation 29}$$

Where *t* is the time required to cool the volume of water and q_c is the cooling rate or wattage of the Peltier cell. By combining the previous two equations, a time dependent equation for the heat transfer in terms of the water properties, volume, temperature change, and Peltier Cell power is derived:

$$t_{Cooling} = \frac{V_{Volume\,Water}\rho_{Water}C_{p_{Water}}\Delta T}{q_c} \qquad \text{Equation 30}$$

The values for the terms in the equation above are given in Appendix A for each of the four amperage tests.

Conclusions

Analysis of the thermal model of the product and four different test amperages revealed an interesting relationship between amperage and cooling power. As the amperage of a Peltier Cell increased, the cooling power and the potential temperature differential that it could create also increased, but the heat lost due to Joule heating also greatly increased. The tested amperages were 3 amps, 4 amps, 5 amps, and 6 amps because at any amperages greater than that, Joule heating dominated heat transfer on the hot side of the Peltier Cell and decreased cooling power.

The data produced by the mathematical model aligns with the data produced experimentally by our prototype. In optimum tests, our prototype was only able to cool a 250 ml water bath by approximately 0.75 °C per minute or 4°C for every 2 minute cooling cycle. Our mathematical model suggests that even without additional losses to the ambient, a 3 amp Peltier Cell would take nearly 21 minutes to cool the water bath 17 °C. The prototype’s performance appears to coincide with theoretical model, so we believe that we can use the model to guide future design decisions.

We had initially believed that the slow cooling time was due to the fact that we could not create a higher amperage circuit necessary to create a larger temperature differential but did not take into account the additional Joule heating that would occur. Had we been able to create a 9 amp circuit and run it with this prototype, we now believe the actual performance of our prototype would have decreased.

The major areas for improvement in our prototype from a thermal analysis perspective are in the design of the heat sink and the number of Peltier Cells used. Improving the design or implementation of the heat sink would more effectively cool the Hot Side of the Peltier Cell and a small temperature differential would be required across the Peltier Cell. The Peltier Performance Chart reveals that a Peltier Cell at any amperage can create higher heat transfer rate as the temperature differential is decreased. By examining Equation 25:

$$T_{Hot} = I^2 R \left(\frac{L_{Copper\ Plate}}{k_{Copper\ Plate} A_{Copper\ Plate}} + \frac{1}{h_{Heat\ Sink} A_{Heat\ Sink}} \right) + T_{Ambient}$$

It is possible to see that given a thin copper plate (i.e. a material with a very high *k* (thermal conductivity) value and a small *L* (thickness)), the first thermal resistor could approach zero and the equation would become highly dependent on the thermal resistance of convective heat transfer.

Another area for improvement would be the number of Peltier Cells used to cool the drink. By increasing the number of Peltier Cells to four, each with their own heat sink, it would be possible to decrease the time required to cool a drink by a factor of four as well. The effect would be dramatic. For example, based on the calculations from our model, four 5 amp Peltier Cells could reduce the cooling time from 15 minutes to less than 4 minutes. This would be a drastic improvement in the product with a minimal amount of new engineering and would allow us to hit our target goals for both time and temperature.

The thermal analysis of our product revealed the areas that would allow us to quickly and efficiently improve our product but also revealed that our prototype and proof of concept matched the theoretical calculations for our product. Further engineering work should be performed on this product to help refine calculation methods, eliminate unnecessary conservatism (such as the Joule heating approximation for Hot Side Peltier energy balance), and improve the overall efficiency of the system.

Appendix A: Variable Values and Calculations

Cooling Channels/Block Reynolds Number and Convection Coefficient Data and Calculations			
Variable	**Value**	**Units**	**Description**
VolChannel	0.00024	m^3/s	Volumetric Flow Rate in the Channel
VisWater	0.000001004	m^2/s	Kinematic Viscosity of Water
PChannel	0.03106	m	Perimeter of the Flow Channel
WChannel	0.0127	m	Width of Flow Channel
DChannel	0.00283	m	Depth of Flow Channel
ReChannel	30784.78103	Unitless	Reynolds Number of Flow in Channel from Equation 5
AChannel	0.000035941	m^2	Area of Flow Channel
PrWater	7.01	Unitless	Prandlt Number of Water
kWater	0.58	W/(m*K)	Thermal Conductivity of Water
hCoolingBlock	20141.48066	W/(m^2*K)	Convective Heat Transfer Coefficient for Cooling Block Calculated from Equation 9

Heat Sink Reynolds Number and Convection Coefficient Data and Calculations			
Variable	**Value**	**Units**	**Description**
VolFan	0.025	m^3/s	Volumetric Flow Rate of Heat Sink Fan
DHeatSink	0.0015	m	Spacing between Heat Sink Plates
VisAir	0.00001511	m^2/s	Kinematic Viscosity of Air
AHeatSink	0.01844	m^2	Available Fan Flow Area
WHeatSink	0.003	m	Characteristic Length of Flow
ReHeatSink	269.1757193	Unitless	Reynolds Number of Flow through Heat Sink Calculated from Equation 5
PrAir	0.713	Unitless	Prandlt Number of Air
kAir	0.0257	W/(m*K)	Thermal Conductivity of Air
LHeatSink	0.08	m	Length of Heat Sink Plate
hHeatSink	3.126504083	W/(m^2*K)	Convective Heat Transfer Coefficient for Heat Sink Calculated from Equation 14

System Values Used in Heat Transfer Rate, Thermal Circuit, and Cooling Time Calculations			
Variable	**Value**	**Units**	**Description**
TAmbient	22	°C	Room Temperature
LCopperPlate	0.00635	m	Thickness of Copper Plate
KCopperPlate	400	W/(m^2*K)	Thermal Conductivity of Copper
ACopperPlate	0.005096	m^2	Surface Area of Copper Plate
ATotalHeatSink	0.674	m^2	Total Surface Area of Heat Sink Plates
R	1.3	ohms	Electical Resistance of the Peltier Cell
VolBev	0.00025	m^3	Volume of 250 ml Drink in m^3
DenWater	1000	kg/m^3	Density of Water
SpeHWater	4813	J/(kg*K)	Specific Heat of Water
TDiff	17	°C	Temperature Change to Cool Beverage from Ambient

Heat Transfer Rate, Thermal Circuit, and Cooling Time Calculations for 3 Amp Peltier Circuit			
Variable	**Value**	**Units**	**Description**
TestAmperage1	3	amps	Peltier Operating Amperage of 3 Amps
THot1	27.58867152	°C	Temperature of Hot Side of Peltier Cell at 3 Amps Using Equation 25
TCold1	4.588671525	°C	Peltier Cold Side Temperature at 3 Amps
TDifferential 1	23	°C	Temperature Difference from THot1 for TCold1 < 5°C
QC1	16	W	Peltier Cold Side Cooling Rate at 3 Amps from Chart
TFluid1	4.744931528	°C	Water Bath Temperatue at 3 Amps Using Equation 27
CoolingTime1	1278.453125	s	Time Required to Cool 250 ml of Beverage by 17°C at 3 Amps

Heat Transfer Rate, Thermal Circuit, and Cooling Time Calculations for 4 Amp Peltier Circuit			
Variable	**Value**	**Units**	**Description**
TestAmperage2	4	amps	Peltier Operating Amperage of 4 Amps
THot2	31.93541604	°C	Temperature of Hot Side of Peltier Cell at 4 Amps Using Equation 25
TCold2	4.935416044	°C	Peltier Cold Side Temperature at 4 Amps
TDifferential 2	27	°C	Temperature Difference from THot2 for TCold2 < 5°C
QC2	20	W	Peltier Cold Side Cooling Rate at 4 Amps from Chart
TFluid2	5.130741049	°C	Water Bath Temperatue at 4 Amps Using Equation 27
CoolingTime2	1022.7625	s	Time Required to Cool 250 ml of Beverage by 17°C at 4 Amps

Heat Transfer Rate, Thermal Circuit, and Cooling Time Calcuations for 5 Amp Peltier Circuit			
Variable	**Value**	**Units**	**Description**
TestAmperage3	5	amps	Peltier Operating Amperage of 5 Amps
THot3	37.52408757	°C	Temperature of Hot Side of Peltier Cell at 5 Amps Using Equation 25
TCold3	4.524087569	°C	Peltier Cold Side Temperature at 5 Amps
TDifferential 3	33	°C	Temperature Difference from THot3 for TCold3 < 5°C
QC3	22	W	Peltier Cold Side Cooling Rate at 5 Amps from Chart
TFluid3	4.738945074	°C	Water Bath Temperatue at 5 Amps Using Equation 27
CoolingTime3	929.7840909	s	Time Required to Cool 250 ml of Beverage by 17°C at 5 Amps

Heat Transfer Rate, Thermal Circuit, and Cooling Time Calculations for 6 Amp Peltier Circuit			
Variable	**Value**	**Units**	**Description**
TestAmperage4	6	amps	Peltier Operating Amperage of 6 Amps
THot4	44.3546861	°C	Temperature of Hot Side of Peltier Cell at 6 Amps Using Equation 25
TCold4	4.3546861	°C	Peltier Cold Side Temperature at 6 Amps
TDifferential 4	40	°C	Temperature Difference from THot4 for TCold4 < 5°C
QC4	19	W	Peltier Cold Side Cooling Rate at 6 Amps from Chart
TFluid4	4.540244854	°C	Water Bath Temperatue at 6 Amps Using Equation 27
CoolingTime4	1076.592105	s	Time Required to Cool 250 ml of Beverage by 17°C at 6 Amps

Referenced Equations and Figures

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