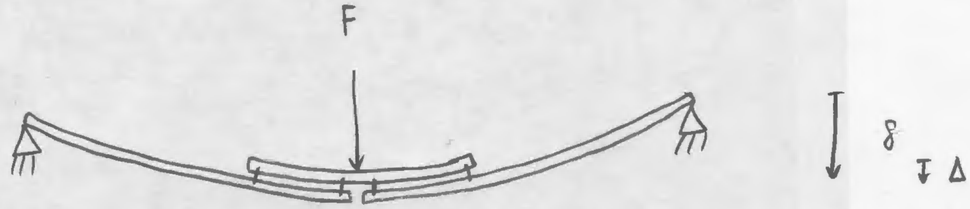
BEAM BENDING EQUATIONS

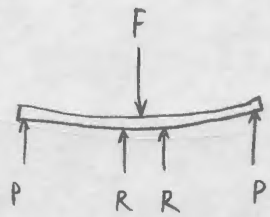
We are solving for the deflexion of the foldable snowboard under an applied load:



In the calculations that follow, δ is the max deflection of the board, while Δ is the deflexion of the center piece.

The center piece is either a plate or two rods. It is connected to each half by two attachments.

There is a force in each of the attachments:



view of center piece

We will need to find those forces when calculating δ. The center piece (either a plate or two rods has a length ℓ. Its properties: Young's Modulus E and a second moment of area I. It is by changing I that we will differentiate the tube design and the plate design.

For the plate design, $I = \dfrac{bh^3}{12}$ (h height of plate)

For the 2-rod design, $I = \dfrac{\pi}{32}\left(D_{out}^4 - D_{in}^4\right)$ (D_{out} is outer diameter, D_{in} inner diameter)

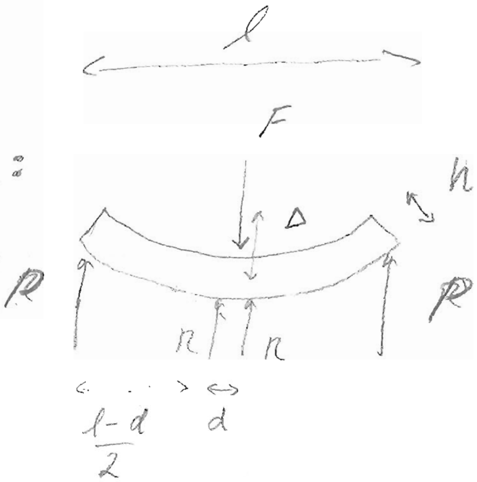
The distance between the two middle attachments is called d.

Each half of the board has a length L, width b, Young's Modulus E and moment of area J.

We find first the deflection of the center piece, the we find the deflection of the left half. Using boundary conditions we can find the value of R and P, and obtain the final equation which gives a condition on the parameters of the center piece such that the deflection δ is the same as for the original board.

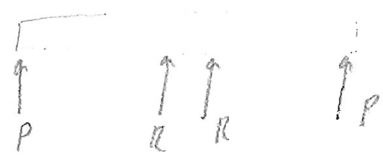
Beam Bending Equations

Plate:



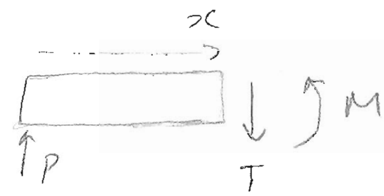
b : width of plate

d, δ and F is known
l and h and E need to be optimized/chosen



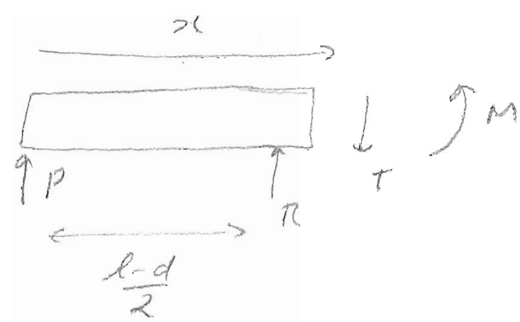
ignore mass of plate

between $0 < x < \frac{l-d}{2}$



$T = P \qquad M = Tx$

between $\frac{l-d}{2} < x < l/2$



$T = P + R \qquad M = Tx - \frac{R}{2}(l-d)$

Ignore the bending due to shear stress (usually contributes very little)

$$y'' = -\frac{M(x)}{EI} = -\frac{Px}{EI} \qquad y' = -\frac{Px^2}{2EI} + C_1$$

$$y(x) = -\frac{Px^3}{6EI} + C_1 x + C_2$$

[Ignore the difference to the deformed equation due to R, consider R applied very close to the center] so we neglect this effect :

$$y(0) = 0 \Rightarrow C_2 = 0 \qquad y'(\ell/2) = 0$$

$$0 = -\frac{P(\ell/2)^2}{2EI} + C_1 \qquad C_1 = \frac{P\ell^2}{8EI}$$

$$\boxed{y(x) = -\frac{Px^3}{6EI} + \frac{P\ell^2}{8EI}x}$$

$$\Delta = -\frac{P(\ell/2)^3}{6EI} + \frac{P\ell^2\,\ell/2}{8EI} \qquad \boxed{\Delta = \frac{P\ell^3}{24EI}}$$

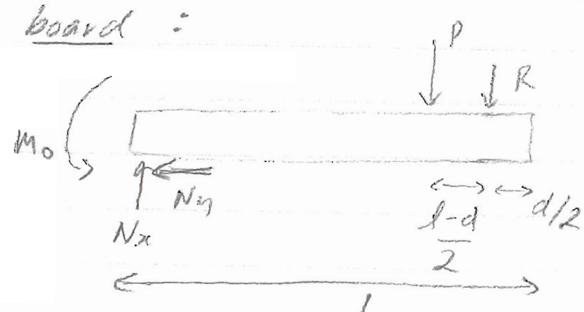
Equilibrium Equations

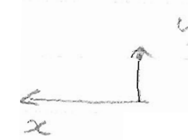
$$\sum \vec{F} = 0 \Rightarrow \boxed{F = 2P + 2R}$$

$$\sum \vec{M}_{end} = 0 \Rightarrow R\left(\frac{\ell-d}{2}\right) + R\left(\frac{\ell+d}{2}\right) + \ell P = \frac{\ell}{2}F$$

$$R\ell + P\ell = \frac{F\ell}{2} \qquad F = 2R + 2P$$

Now consider the board :

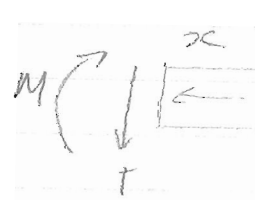




between 0 and $d/2$: free

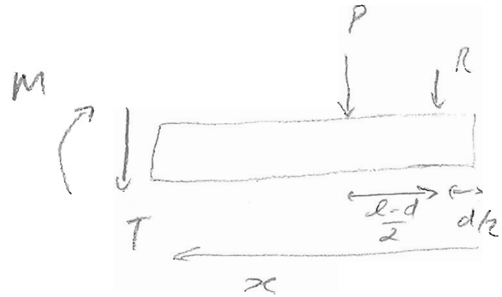
between $d/2 < x < \ell/2$

$$T = -R \qquad M = Tx + \frac{Rd}{2}$$



$$M = -Rx + Rd/2$$

between $\frac{l}{2} < x < L$



$$T = -P - R$$

$$M = Tx + \frac{l}{2}P + \frac{d}{2}R = (-P-R)x + \frac{l}{2}P + \frac{d}{2}R$$

Now find the deflected shape between the clamp and the plate (again ignore bending due to shear stress)

$$y_2'' = -\frac{M(x)}{\Sigma J}$$ where Σ, J corresponds to E, I but for the snowboard

$$y_2'' = \frac{(P+R)x - \frac{l}{2}P - \frac{d}{2}R}{\Sigma J}$$

$$y_2' = \frac{(P+R)x^2}{2\Sigma J} - \frac{(lP+dR)x}{2\Sigma J} + D_1$$

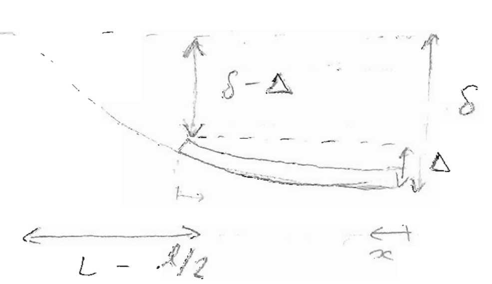
$$y_2 = \frac{(P+R)x^3}{6\Sigma J} - \frac{(lP+dR)x^2}{4\Sigma J} + D_1 x + D_2$$

$$y_2(L) = 0 \Rightarrow 0 = \frac{(P+R)L^3}{6\Sigma J} - \frac{(lP+dR)L^2}{4\Sigma J} + D_1 L + D_2$$

$$y_2(l/2) = \delta - \Delta$$

A

$0 = f(E, l, h \quad)$



$\delta(F) =$

(B). $$\delta - \Delta = \frac{(P+R)(l/2)^3}{6EJ} - \frac{(lP + dR)(l/2)^2}{4EJ} + D_1\, l/2 + D_2$$

once D_1, D_2 are found (A), (B) can be used to simplify P, R

$$\delta - \Delta = \frac{(P+R)[l^3/8 - L^3]}{6EJ} - \frac{(lP + dR)(l^2/4 - L^2)}{4EJ} + D_1\left(\frac{l}{2} - L\right)$$

Another property is that $y_1'(0) = -y_2'(l/2)$

$$\boxed{\frac{Pl^2}{8EI} = -\frac{(P+R)(l/2)^2}{2EJ} + \frac{(lP + dR)(l/2)}{2EJ} - D_1}$$

Last find the deformed shape of the board from $0 < x < l/2$

$$y_3'' = -\frac{M(x)}{EJ} = \frac{Rx - Rd/2}{EJ}$$

$$y_3' = \frac{R\frac{x^2}{2} - Rdx/2}{EJ} + y_0 = \frac{Rx^2}{2EJ} - \frac{Rdx}{2EJ} + y_0$$

$$y_3 = \frac{Rx^3}{6EJ} - \frac{Rdx^2}{4EJ} + y_0 x + y_1$$

$y_3(0) = \delta = y_1 \qquad y'(0) = 0 \Rightarrow y_0 = 0$

$$\boxed{y_3 = \frac{Rx^3}{6EJ} - \frac{Rdx^3}{4EJ} + \delta}$$

$$y_3(l/2) = y_2(l/2) \qquad y_3'(l/2) = y_2'(l/2)$$

$$\cancel{\frac{R(l/2)^3}{6EJ}} - \cancel{\frac{Rd(l/2)^2}{4EJ}} + \delta = \frac{(P+\cancel{R})(l/2)^3}{6EJ} - \frac{(lP+\cancel{dR})(l/2)^2}{4EJ} + D_1\, l/2 + D_2$$

and

$$\frac{R(l/2)^2}{2EJ} - \frac{Rd(l/2)}{2EJ} = \frac{(P+\cancel{R})(l/2)^2}{2EJ} - \frac{(lP+\cancel{dR})(l/2)}{2EJ} + D_1$$

$$\frac{Pl^2}{4EJ} - \frac{Pl^2}{8EJ} = \boxed{D_1 = \frac{Pl^2}{8EJ}}$$

$$\frac{Pl^3}{16EJ} - \frac{Pl^3}{48EJ} - \frac{Pl^3}{16EJ} + \delta = D_2$$

$$\boxed{D_2 = -\frac{Pl^3}{48EJ} + \delta}$$

$$\boxed{y_2 = \frac{(P+R)x^3}{6EJ} - \frac{(lP+Rd)x^2}{4EJ} + \frac{Pl^2}{8EJ}x - \frac{Pl^3}{48EJ} + \delta}$$

$$F = 2P + 2R \quad \Rightarrow \quad 2R = F - 2P$$

$$D_1 = \frac{Pl^2}{8EJ} \qquad D_2 = -\frac{Pl^3}{48EJ} + \delta$$

$$y_2'(L) = \frac{(P+R)L^2}{2EJ} - \frac{(lP + dR)L}{2EJ} + \frac{Pl^2}{8EJ} = 0$$

$$\frac{1}{8EJ}\left[4(P+R)L^2 - 4(lP+dR)L + Pl^2\right] = 0$$

$$4PL^2 + 4RL^2 - 4lPL - 4dRL + Pl^2 = 0$$

$$4PL^2 + 2(F-2P)L^2 - 4lPL - 2(F-2P)dL + Pl^2 = 0$$

$$\cancel{4PL^2} + 2FL^2 - \cancel{4PL^2} - 4lLP - 2FdL + 4PdL + Pl^2 = 0$$

$$P(-4lL + 4dL + l^2) = -2FL^2 + 2FdL$$

$$\boxed{P = \frac{2FL(L-d)}{4Ll - 4dL - l^2}}$$

$$\boxed{R = \frac{F}{2} - \frac{2FL(L-d)}{4Ll - 4dL - l^2}}$$

$$y_2(\ell/2) = \frac{(P+R)\ell^3}{48EJ} - \frac{(\ell P + Rd)\ell^2}{16EJ} + \frac{P\ell^3}{16EJ} - \frac{P\ell^3}{48EJ} + \cancel{\delta} = \cancel{\delta} - \frac{P\ell^3}{24EI}$$

$$\frac{(P+R)\ell^3}{48EJ} - \frac{(\ell P + Rd)\ell^2}{16EJ} + \frac{P\ell^3}{24EJ} + \frac{P\ell^3}{24EI} = 0$$

$$P\left(\cancel{\frac{\ell^3}{48EJ}} - \cancel{\frac{\ell^3}{16EJ}} + \cancel{\frac{\ell^3}{24EJ}} + \frac{\ell^3}{24EI}\right) + R\left(\frac{\ell^3}{48EJ} - \frac{d\ell^2}{16EJ}\right) = 0$$

$$\frac{2L(L-d)}{4L\ell - 4dL - \ell^2}\,\frac{\ell}{24EI} + \left(\frac{1}{2} - \frac{2L(L-d)}{4L\ell - 4dL - \ell^2}\right)\frac{\ell - 3d}{48EJ} = 0$$

$$\frac{4L(L-d)\ell}{24EI} + \left(4L\ell - \cancel{4dL} - \ell^2 - 4L(L - \cancel{d})\right)\frac{\ell - 3d}{48EJ} = 0$$

$$\boxed{\frac{4L(L-d)\ell}{EI} + \frac{(4L\ell - \ell^2 - 4L^2)(\ell - 3d)}{2EJ} = 0}$$